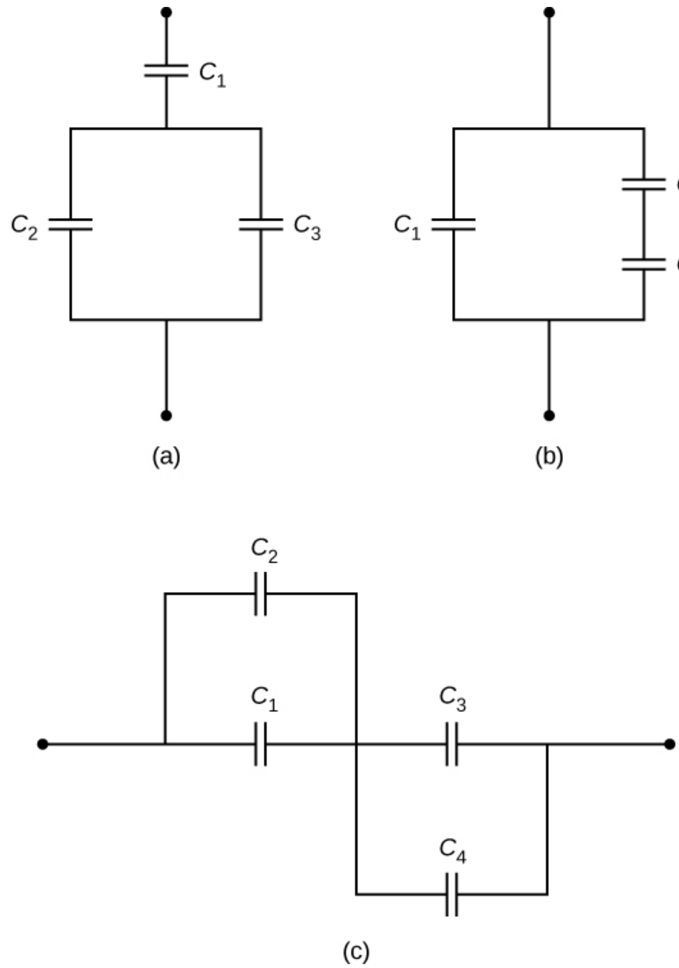
8.5 Check Your Understanding Determine the net capacitance C of each network of capacitors shown below. Assume that $C_1 = 1.0\,\text{pF}$, $C_2 = 2.0\,\text{pF}$, $C_3 = 4.0\,\text{pF}$, and $C_4 = 5.0\,\text{pF}$. Find the charge on each capacitor, assuming there is a potential difference of 12.0 V across each network.



8.3 | Energy Stored in a Capacitor

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy is stored in a capacitor
- Use energy relations to determine the energy stored in a capacitor network

Most of us have seen dramatizations of medical personnel using a defibrillator to pass an electrical current through a patient's heart to get it to beat normally. Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics to supply energy when batteries are charged (**Figure 8.15**). Capacitors are also used to supply energy for flash lamps on cameras.

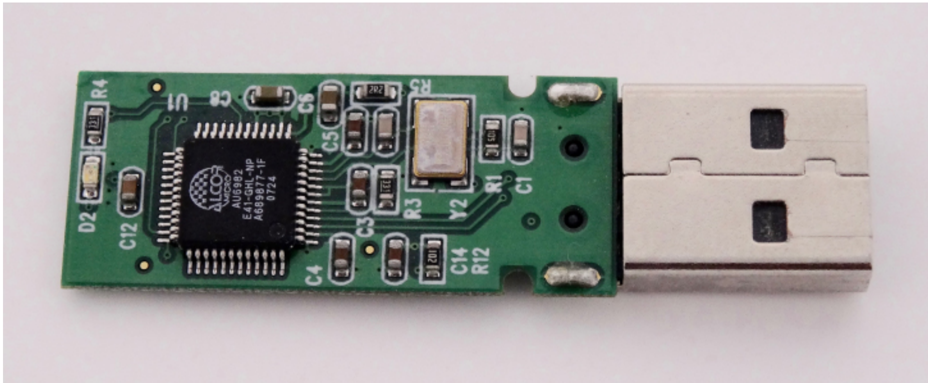


Figure 8.15 The capacitors on the circuit board for an electronic device follow a labeling convention that identifies each one with a code that begins with the letter “C.” (credit: Windell Oskay)

The energy U_C stored in a capacitor is electrostatic potential energy and is thus related to the charge Q and voltage V between the capacitor plates. A charged capacitor stores energy in the electrical field between its plates. As the capacitor is being charged, the electrical field builds up. When a charged capacitor is disconnected from a battery, its energy remains in the field in the space between its plates.

To gain insight into how this energy may be expressed (in terms of Q and V), consider a charged, empty, parallel-plate capacitor; that is, a capacitor without a dielectric but with a vacuum between its plates. The space between its plates has a volume Ad, and it is filled with a uniform electrostatic field E. The total energy U_C of the capacitor is contained within this space. The **energy density** u_E in this space is simply U_C divided by the volume Ad. If we know the energy density, the energy can be found as $U_C = u_E(Ad)$. We will learn in **Electromagnetic Waves** (after completing the study of Maxwell’s equations) that the energy density u_E in a region of free space occupied by an electrical field E depends only on the magnitude of the field and is

$$u_E = \frac{1}{2}\varepsilon_0 E^2. \tag{8.9}$$

If we multiply the energy density by the volume between the plates, we obtain the amount of energy stored between the plates of a parallel-plate capacitor: $U_C = u_E(Ad) = \frac{1}{2}\varepsilon_0 E^2 Ad = \frac{1}{2}\varepsilon_0 \frac{V^2}{d^2} Ad = \frac{1}{2}V^2 \varepsilon_0 \frac{A}{d} = \frac{1}{2}V^2 C$.

In this derivation, we used the fact that the electrical field between the plates is uniform so that $E = V/d$ and $C = \varepsilon_0 A/d$. Because $C = Q/V$, we can express this result in other equivalent forms:

$$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV. \tag{8.10}$$

The expression in **Equation 8.10** for the energy stored in a parallel-plate capacitor is generally valid for all types of capacitors. To see this, consider any uncharged capacitor (not necessarily a parallel-plate type). At some instant, we connect it across a battery, giving it a potential difference $V = q/C$ between its plates. Initially, the charge on the plates is $Q = 0$. As the capacitor is being charged, the charge gradually builds up on its plates, and after some time, it reaches the value Q. To move an infinitesimal charge dq from the negative plate to the positive plate (from a lower to a higher potential), the amount of work dW that must be done on dq is $dW = V dq = \frac{q}{C} dq$.

This work becomes the energy stored in the electrical field of the capacitor. In order to charge the capacitor to a charge Q, the total work required is

$$W = \int_0^{W(Q)} dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}.$$

Since the geometry of the capacitor has not been specified, this equation holds for any type of capacitor. The total work W needed to charge a capacitor is the electrical potential energy U_C stored in it, or $U_C = W$. When the charge is expressed in coulombs, potential is expressed in volts, and the capacitance is expressed in farads, this relation gives the energy in joules.

Knowing that the energy stored in a capacitor is $U_C = Q^2/(2C)$, we can now find the energy density u_E stored in a vacuum between the plates of a charged parallel-plate capacitor. We just have to divide U_C by the volume Ad of space between its plates and take into account that for a parallel-plate capacitor, we have $E = \sigma/\varepsilon_0$ and $C = \varepsilon_0 A/d$. Therefore, we obtain

$$u_E = \frac{U_C}{Ad} = \frac{1}{2} \frac{Q^2}{C} \frac{1}{Ad} = \frac{1}{2} \frac{Q^2}{\varepsilon_0 A/d} \frac{1}{Ad} = \frac{1}{2} \frac{1}{\varepsilon_0} \left(\frac{Q}{A}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} = \frac{(E\varepsilon_0)^2}{2\varepsilon_0} = \frac{\varepsilon_0}{2} E^2.$$

We see that this expression for the density of energy stored in a parallel-plate capacitor is in accordance with the general relation expressed in **Equation 8.9**. We could repeat this calculation for either a spherical capacitor or a cylindrical capacitor—or other capacitors—and in all cases, we would end up with the general relation given by **Equation 8.9**.

Example 8.8

Energy Stored in a Capacitor

Calculate the energy stored in the capacitor network in **Figure 8.14**(a) when the capacitors are fully charged and when the capacitances are $C_1 = 12.0\,\mu\text{F}$, $C_2 = 2.0\,\mu\text{F}$, and $C_3 = 4.0\,\mu\text{F}$, respectively.

Strategy

We use **Equation 8.10** to find the energy U_1, U_2, and U_3 stored in capacitors 1, 2, and 3, respectively. The total energy is the sum of all these energies.

Solution

We identify $C_1 = 12.0\,\mu\text{F}$ and $V_1 = 4.0\,\text{V}$, $C_2 = 2.0\,\mu\text{F}$ and $V_2 = 8.0\,\text{V}$, $C_3 = 4.0\,\mu\text{F}$ and $V_3 = 8.0\,\text{V}$. The energies stored in these capacitors are

$$\begin{aligned} U_1 &= \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(12.0\,\mu\text{F})(4.0\,\text{V})^2 = 96\,\mu\text{J}, \\ U_2 &= \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(2.0\,\mu\text{F})(8.0\,\text{V})^2 = 64\,\mu\text{J}, \\ U_3 &= \frac{1}{2} C_3 V_3^2 = \frac{1}{2}(4.0\,\mu\text{F})(8.0\,\text{V})^2 = 130\,\mu\text{J}. \end{aligned}$$

The total energy stored in this network is

$$U_C = U_1 + U_2 + U_3 = 96\,\mu\text{J} + 64\,\mu\text{J} + 130\,\mu\text{J} = 0.29\,\text{mJ}.$$

Significance

We can verify this result by calculating the energy stored in the single $4.0\text{-}\mu\text{F}$ capacitor, which is found to be equivalent to the entire network. The voltage across the network is 12.0 V. The total energy obtained in this way agrees with our previously obtained result, $U_C = \frac{1}{2}CV^2 = \frac{1}{2}(4.0\,\mu\text{F})(12.0\,\text{V})^2 = 0.29\,\text{mJ}$.

8.6 Check Your Understanding The potential difference across a 5.0-pF capacitor is 0.40 V. (a) What is the energy stored in this capacitor? (b) The potential difference is now increased to 1.20 V. By what factor is the stored energy increased?

In a cardiac emergency, a portable electronic device known as an automated external defibrillator (AED) can be a lifesaver. A **defibrillator** (**Figure 8.16**) delivers a large charge in a short burst, or a shock, to a person's heart to correct abnormal heart rhythm (an arrhythmia). A heart attack can arise from the onset of fast, irregular beating of the heart—called cardiac or ventricular fibrillation. Applying a large shock of electrical energy can terminate the arrhythmia and allow the body's natural pacemaker to resume its normal rhythm. Today, it is common for ambulances to carry AEDs. AEDs are also found in many public places. These are designed to be used by lay persons. The device automatically diagnoses the patient's heart rhythm and then applies the shock with appropriate energy and waveform. CPR (cardiopulmonary resuscitation) is recommended in many cases before using a defibrillator.

Figure 8.16 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies)

Example 8.9

Capacitance of a Heart Defibrillator

A heart defibrillator delivers $4.00 \times 10^2\,\text{J}$ of energy by discharging a capacitor initially at $1.00 \times 10^4\ \text{V}$. What is its capacitance?

Strategy

We are given U_C and V, and we are asked to find the capacitance C. We solve **Equation 8.10** for C and substitute.

Solution

Solving this expression for C and entering the given values yields $C = 2\dfrac{U_C}{V^2} = 2\dfrac{4.00 \times 10^2\ \text{J}}{(1.00 \times 10^4\ \text{V})^2} = 8.00\,\mu\text{F}.$

8.4 | Capacitor with a Dielectric

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects a dielectric in a capacitor has on capacitance and other properties
- Calculate the capacitance of a capacitor containing a dielectric

As we discussed earlier, an insulating material placed between the plates of a capacitor is called a dielectric. Inserting a dielectric between the plates of a capacitor affects its capacitance. To see why, let's consider an experiment described in **Figure 8.17**. Initially, a capacitor with capacitance C_0 when there is air between its plates is charged by a battery to voltage V_0. When the capacitor is fully charged, the battery is disconnected. A charge Q_0 then resides on the plates, and the potential difference between the plates is measured to be V_0. Now, suppose we insert a dielectric that *totally* fills the gap between the plates. If we monitor the voltage, we find that the voltmeter reading has dropped to a *smaller* value *V*. We write this new voltage value as a fraction of the original voltage V_0, with a positive number κ, $\kappa > 1$:

$$V = \frac{1}{\kappa} V_0.$$

The constant κ in this equation is called the **dielectric constant** of the material between the plates, and its value is characteristic for the material. A detailed explanation for why the dielectric reduces the voltage is given in the next section. Different materials have different dielectric constants (a table of values for typical materials is provided in the next section). Once the battery becomes disconnected, there is no path for a charge to flow to the battery from the capacitor plates. Hence, the insertion of the dielectric has no effect on the charge on the plate, which remains at a value of Q_0. Therefore, we find that the capacitance of the capacitor with a dielectric is

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \kappa \frac{Q_0}{V_0} = \kappa C_0. \tag{8.11}$$

This equation tells us that the *capacitance* C_0 *of an empty (vacuum) capacitor can be increased by a factor of* κ *when we insert a dielectric material to completely fill the space between its plates*. Note that **Equation 8.11** can also be used for an empty capacitor by setting $\kappa = 1$. In other words, we can say that the dielectric constant of the vacuum is 1, which is a reference value.